Teaching Learners with Dyslexia

Mathematics

10 + 15 = 25

22 - 8 = 14

34 * 8 = 272

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Inclusion in Europe through Knowledge and Technology

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Teaching Mathematics to Students who have Dyslexia

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Inclusion in Europe through knowledge and technology

Information on the fundamental principles, practices, educational material and teaching aids used to teach various subjects to students with special needs are few and far between. In some cases, material has been prepared for internal use at specialised schools or in other closed environments. In other cases, knowledge has been passed from teacher to teacher as part of workplace training.

No systematic material on pedagogical principles, practices, educational material and teaching aids exist for areas such as teaching first language teaching, foreign language teaching, mathematics and music for the blind.

With this in mind, the goal of this European project is to further develop, implement and disseminate good practices in the area of inclusive education and learning technologies by delivering three primary components: *Teaching Guides*, Guide on g*ood practices Inclusive learning and Teaching* and *SMART E-learning objects*.

Teaching guides

In completing the project, RoboBraille partners have created a series of twelve educational guides covering fundamental principles, practices, educational material and teaching aids covering first language teaching, foreign language teaching, mathematics and music for the blind, partially sighted and learners with dyslexia.

Inclusion guide on good practices for inclusive learning and teaching

In support of this, the project has collected and collated information on good inclusion practices in five select areas (teacher skills, alternate media, support structures, preparation for inclusion and teaching environments) which are published in a catalogue of good practices.

SMART E-learning

Finally, the project will adapt a comprehensive set of educational material on the RoboBraille service prepared in the LLL LdV RoboBraille SMART project into a set of learning objects for popular e-learning platforms for web and tablet deployment.

For all materials produced by this project

Because the material covers teaching of students of various age, they are named students, learners, pupils and children. The material also reflects the different culture and level of inclusion practices of the project partners. The guide is not a substitute for formal training of teachers.

Introduction to this teaching guide

Dyslexia is by now a widely known but poorly understood specific learning disability. It can be difficult to define because the causes underlying its measurable manifestations can be very variable. However, dyslexia is a real problem, which affects the learning of reading and writing of many individuals and whose effects may be exacerbated by an inadequate education. The complexity of the problem is increased by the fact that dyslexia and reading and writing difficulties may vary according to the cultural and linguistic background.

There are several definitions of dyslexia but probably the most frequently used on is the definition introduced by the International Dyslexia Association (IDA) in 2002: "Dyslexia is a specific learning disability (SpLD) that is neurobiological in origin. It is characterized by difficulties with accurate and/or fluent word recognition and by poor spelling and decoding abilities. These difficulties typically result from a deficit in the phonological component of language that is often unexpected in relation to other cognitive abilities and the provision of effective classroom instruction. Secondary consequences may include problems in reading comprehension and reduced reading experience that can impede growth of vocabulary and background knowledge."

Dyslexia is a hidden disability thought to affect around 10% of the population, 4% severely. It is the most common of the SpLDs.

It is essential to keep in mind if we want to understand the problem of learners with dyslexia, that the most frequent type of dyslexia is caused not only by phonological difficulties. The difference of these learners' way of learning is an overall speciality in the information processing. Problems may arise in non-literal areas, like math, too. The best math teaching practices can help to avoid difficulties in math and help to be successful in the subject not only for learners with dyslexia, but for all learners.

This section provides insight of the similarities and differences in dyslexia and dyscalculia, a difficulty in learning or comprehending arithmetic and learning facts in mathematics. Clarification of the concepts can lead to a more directed intervention.

The teaching and learning methods that are suggested in this guide are methods proven to be efficient teaching and learning methods, and ground an inclusive teaching environment.

Specialized pedagogies for teaching mathematics to students with dyslexia

40-60 % of students with dyslexia also suffer from dyscalculia. It is difficult to estimate the rate because the incidence depends on the teaching methods. Proper proceedings can prevent the emergence of dyscalculia in students with dyslexia.

Learners with dyslexia can be successful in any subject area using an instruction that is adapted to their individual way of thinking. The learning happens not only when there is teaching. Moreover, the process is often more effective without constant instructions.

Levels of learning and the learner with dyslexia

The natural learning starts with free exploration, not with rules.

The human brain works on three levels, with three types of learning:

- Overall exploration.
- Step by step procession, naming and using rules.
- Holistic understanding.

The learner does not need any sequential processing at the first level. It is merely to get acquainted with the subject. If teaching starts with free exploration, the learner can build a connection to the material without any pressure to be good or to achieve. It means that there will be less anxiety, so the interest may help to continue the learning and put more effort into the tas .

The dyslexic brain is perfect for that type of learning, so it is an explicit advantage for learners with dyslexia, while all learners can benefit from an anxiety free learning situation.

The challenge for the dyslexic brain is the second type of learning, when knowledge should become more systematic. The concepts should be clarified, clear rules and a lot of methodical practise can lead to safe learning. This is the type of learning that has to be facilitated by a proper teaching method for any learners, but especially for learners with dyslexia.

We are going to give methodological guidance and examples on how this can be achieved, while the best support for the learners with dyslexia is to start any learning with overall exploration.

The third level of learning is the type of learning that cannot be taught, but rather fostered. An own vision of the material is the highest level of knowledge. One needs the overall sense of the topic and the stable rules and practical knowledge to create such an insight, the holistic knowledge. The third level happens only in the learner's mind after getting through the other two levels.

Learners with dyslexia have an advantage at this level, as the holistic information processing is their strength. That means that if a learner with dyslexia can solve the middle level of the learning process, the highest math achievement levels are open.

There are several mathematicians with poor calculation but great math skills, for example the former researcher of the IBM, the developer of the fractal geometry, Benoit Mandelbrot.

The dyslexic brain is not better or worse than other, it is just more predisposed to work holistically, and less adaptable to stimuli which can cause difficulties and advantages in any cognitive areas. To get the best of these "special brains" we have to understand them.

Types of dyslexia and the connection to a type of dyscalculia

Although difficulties with numeracy and maths are common with dyslexia, the connection, similarities and differences of the two problem areas are less understood.

The most common form of dyslexia is the difficulty in learning to read. This can lead to slow reading and poor spelling later in adulthood, but does not hinder basically the reading ability if proper teaching helps the learner. There are many persons with dyslexia showing high level literal achievements (e.g., the Irish poet William Butler Yeats) and even more who are avid readers.

However, there is a rare form of dyslexia, the form when the person cannot match sounds to letters. It is called 'deep dyslexia'[1], while the above-mentioned type is the 'surface dyslexia'.

The type of dyslexia depends of the stage where acquisition of reading skills has been stuck. There are three stages of the development of the reading skills (Frith, 1985):

1. At the logographic stage, as the term indicates, words are recognised and stored as images. There is no letter-to-sound correspondence, yet, only the whole picture of the word. A person with deep/severe/phonological/ dyslexia gets stuck at the logographic stage, and is unable to match a sound to the corresponding letter, because the phonological module (Fodor, 1983) is functioning inadequately. These individuals are unable to read out non-sense texts, and only know words that they have seen and their memory has kept logographically like an image.
2. The alphabetic reading stage is where grapheme-phoneme correspondences appear. The learner is able to spell out words letter-by-letter at this stage. In the case of surface dyslexia, the letters and words and their sequences are not stable, and that way the words cannot be kept in the mental lexicon. The phonological route is intact, and the individual is capable of matching letters to sounds and reading out any text, even nonsensical words, but only by spelling them out. The images of whole words are not stored in their mental lexicon.

[1] Often called severe or phonological dyslexia

3. At the orthographic stage, the learner, armed with phonological knowledge, is able to read words and sentences without spelling them out using the images of the words in the mental lexicon.

An example: Depending on what kind of reading a learner is capable of, the relevant reading stage can be determined.

Test text:

Aoccdrnig to a rscheearch at an Elingsh uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is that the frist and lsat ltteer is at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae we do not raed ervey lteter by it slef but the wrod as a wlohe.

- The above text can only be read by those at the orthographic stage of reading.
- An individual at the alphabetic stage will read nonsensical groups of letters.
- An individual at the logographic level is unable to spell out even the nonsensical groups of letters.

The prevalence of dyslexia is about 15-20 %, while in the case of deep/sever/phonological dyslexia, prevalence is only 3-4 %. As dyslexia is not a disease, but perhaps rather a special way of information processing, and it causes difficulties, the signs are not unequivocal in many cases, and therefore the estimation of prevalence is uncertain.

Deep dyslexia is more recognizable since insufficient phonological abilities can be identified easily. Unfortunately, even the best teaching methods cannot help in these cases to achieve usable reading skills. Hence for people with deep dyslexia, assistive technologies offer the best accommodation for inclusion.

Dyscalculia can appear in the case of surface dyslexia if the math teaching instruction is not helping the holistic way of information processing that is one of the main characteristics of the dyslexic brain. The same neurological abnormalities that cause surface dyslexia can cause a type of dyscalculia, while there are types of dyscalculia that are based on different neurological deviances.

Characteristics of the dyscalculia

The more severe form of dyscalculia is characterized by an inability to understand simple number concepts and to master basic numeracy skills. There are likely to be difficulties dealing with numbers at very elementary levels; these include:

- Learning numerical facts (e.g., important dates) and procedures.
- Telling the time, time keeping.
- Understanding quantity, prices and value of money.

However, the more frequent form of dyscalculia can be circumvented by proper teaching. If the basic number sense is functioning, the problem emerges only as an unstable sense of relations and spatial-orientation, unclear details. These difficulties are similar to those caused by the more common type of dyslexia, the surface dyslexia. The consequence is poor calculation and math abilities, causing extra burdens on learners with dyslexia.

The most characteristic issues in math for a learner with dyslexia include:

- Mixing numbers, quantities, math signs.
- Mixing operations.
- Confusion in sequential, spatial-orientation relations.
- Misunderstanding and mixing math concepts.
- Misunderstanding tasks.

If the basic number concept is intact, the difficulties can be decreased significantly by remedial training or as the best solution, provide proper teaching from the beginning.

Challenges relating to the specific learning difficulty

Every learner is different, and everybody has characteristics that define the best way of their indivídual learning and the best way they can be taught. Learners with dyslexia are challenged, because they are more sensitive to the learning environment. Problems found in the learning environment are therefore often reinforced by this sensitivity.

Teaching learners with dyslexia means to cope with diversity, because the learners are not homogeneous, and the difficulties rooted from different characteristics. Learners with dyslexia differ not only from the majority, but also from each other. Thus, personalized teaching is more a need than a possibility in their case.

Weaknesses and assessment

The following areas can be referred to in the identification of math weaknesses:

- Spatial and temporal orientation.
- Continuity of counting.
- Understanding place values.
- Understanding quantities.
- Operations on numbers: addition, subtraction, multiplication, division.
- Solving contextualised tasks, separately for each type of operation.
- Understanding mathematical-logical rules.
- Number memory.

Through exercises it can be established whether the learner:

- Understands and is able to use the language of mathematics; and
- Understands mathematical symbols.

The counting strategy of the learner can be identified:

- The learner has a strategy, but the method is ineffective
- The learner has a strategy, and is already automatic in some respects.
- The learner has a good counting strategy, but is slow and uncertain.

All these functions are rooted in some basic sub-skills. It is therefore important to establish the level of all of these sub-skills.

Counting difficulties can differ depending on particular sub-skill weaknesses. Therefore, the treatment of these difficulties, not unlike that of dyslexia, should begin by charting the individual's structure of abilities and identifying strengths and weaknesses.

A description of suitable teaching methods and practices

The best teaching methods are those that use as many modalities as possible to process the information, while give way to the exploration and provide strict rules to give safe basis for the knowledge.

Two groups of aids should be considered for the inclusive teaching:

- Visual and verbal aids.
- Experience and estimation.

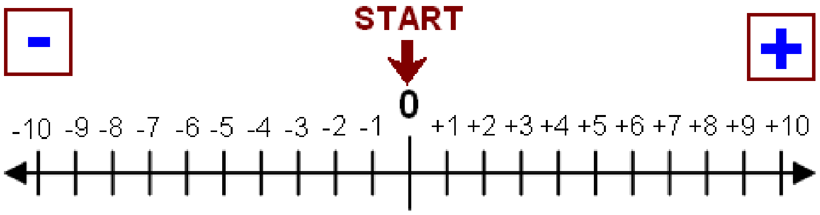
Visual and verbal aids

Visual representation is the first ‘brain tool’ for good maths. Especially if a learner is a ‘right brain person’ with a right brain hemisphere dominance, calculation should be supported by visual tools.

The difficulty in counting is often caused by a poor sense of details and relations. Math concepts have to be clear if one wants to be able to do math. The lack of information on details and relations endangers clarity. One cannot think with vague concepts. When dealing with mathematical expressions, concepts or performing operations, it may help to draw these and explain them with everyday language.

Example 1

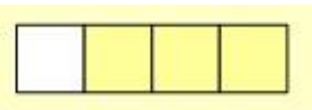
A number line helps counting. It is visual, and it shows the very important point: zero. This is the starting point, and the negative and positive numbers run in opposite directions to the infinitive.



Teaching to count could use this simple tool. That would improve the maths sense. A learner can use the number line to help to sense the numbers.

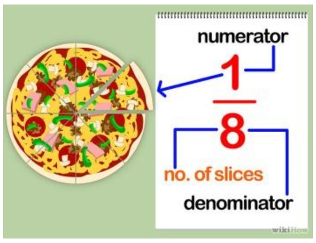
Example 2

A fraction means that something is fractured, broken into pieces. The denominator at the bottom denominates. It tells how many pieces the object was broken into. (Here it is four.)

¾ = 

One can count up in the nominator the number of pieces that should be taken. (Here it is 3.)

This means that a whole has been broken into four identical parts and taken three parts out of the four. All fractions and other maths concepts can be visualized likewise.

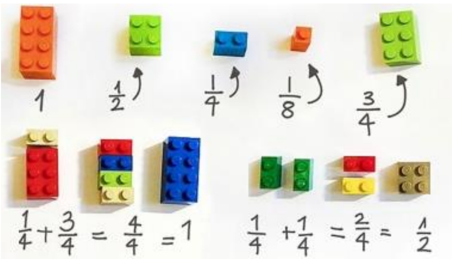


A logical and understandable way to write fractions bottom-up:

1. The nominator is above to tell how many pieces has been taken from the whole.
2. Then there is the fraction line.
3. The denominator is at the bottom.

Example 3

Using LEGO blocks is a method that can be used for any age level to visualise and verbalise a lot of maths concepts. Here is an example on how to get practice in fractions:



Experience and estimation

The human brain works best on real-life experiences. Maths can be seen as a pattern and the learner may be encouraged to find the patterns and experience maths in a practical way.

The use of common sense, estimation and experience can protect against greater mistakes.

Example 4

A scale can be remembered if the learner has clear knowledge of the words:

- “deca” means ten times - 10x - one zero
- “kilo” means thousand times - 1000x - three zeros

Deca means: ten times in any measure. Kilo means: thousand times in any measure.

Estimation means we just perceive the pattern. We take things roughly, and do not deal with the details.

Link amounts to graphical, visual material. One can get a lot of experience during shopping, but it is more beneficial if one can link amounts and units to particular events and objects.

Example 5

1 kilogram bread = 1000 grams: 

10 decagram of butter = 100 gram: 

The learner can remember what the amount looks like.

Other supporting measures: a litre of milk, half a litre of beer, a cent of brandy. These experiences lead to a good base and a guideline to calculate through estimation. The best result can be achieved if the learner establishes his/her own estimation tools.

Learning technologies for inclusive teaching of mathematics to students with dyslexia

Any tool is better than getting lost in counting. One can count using

- Fingers.
- The dial of an analogue watch.
- A ruler.
- Or any other everyday objects.

Beyond these simple tools there are ancient and new technologies that can be used for everybody while they reduce difficulties experienced by learners with dyslexia.

"Everyday tools" model

The everyday tools method is to use anything that needs calculation in the everyday life. Pencil counting, counting money, debt, profit, purchase offers, matchboxes, thermometer and card games are the most used elements, because they are part of the everyday life. While the operation with numbers is difficult in a textbook without real reason and tools, when a learner has to think in pocket money, or count the matchboxes, points or money in a card game, the numbers become meaningful. The tools should be carefully chosen and match the learner's interest.

There are some old methods renewed for the 21st century, and these methods are perfect for the learner with dyslexia.

Chess

The chess board is a great area to use as calculation aid. It visualizes the first 64 numbers. It is a great visual multiplication table as the letters have number values, too (a=1, b=2, etc.).

What is more, the pieces have their values; thus, Chess can be used as a great visualized tool for basic and more advanced math learning (diagrams, etc.).

Judit Polgar, the successful chess champion, has built her educational method on Chess. Children learn to count, read and write through chess and its rules and tools. See:

- Educational guide to the Chess Palace method: http://sakkpalota.hu/index.php/en/for-schools/educational-video
- Description and explanation to the Chess palace method: http://www.slideshare.net/gyarme/chesspalacepolgar-gyarmathy-36590444

Soroban

Soroban is a more than 450-year-old Chinese-Japanese tool which is still in use. Its effectiveness is well illustrated by the fact that though after the second world war, Soroban instruction stopped in Japan, but only temporarily, because according to studies, the mathematical performance of children following the termination of Soroban instruction declined.

Counting on the Soroban makes practice easier and the representation of numbers, as well as doing operations more visual. The use of a Soroban can be particularly useful primarily at the time of laying the foundations of counting.

It exploits both the motor and the visual system in counting. Learners are active and their attention is focussed on the activity with the Soroban. With sufficient practice, all learners are able to detach themselves from the tool and perform the tasks through the movements that have been fixed in their mind.

The Soroban presents an opportunity to acquire diverse counting techniques. The properties of different operations and the relationships between operations can be closely observed during its use. It can provide substantial help for anybody, but especially learners with dyslexia.

Technological tools

Technological tools such as a computer can be a good solution to prevent falling behind. Poorly counting students can be able to perform higher level mathematical tasks, because the machine will take over the burden of counting. Nevertheless, the use of a computer does not mean the translation of the tasks into everyday language or the control of common sense and estimation could be eliminated. Assistive technologies help learners with dyslexia also in maths difficulties.

Calculators

Technical facilities, like a calculator will be helpful even if one has no problem with maths and can use a lot expedient. The digital age helps us with technical devices to do maths, too.

Calculators are around us; thus, it is the easiest way to calculate a sum by pushing the buttons. A smart calculator can solve even quadratic equations.

Anybody can master everyday maths using the technical tools. However technical tools should be only aids, and not a replacement for reasoning.

Programs, applications

There are tremendous useful programs and apps to visualize and solve math problems. Technical devices like smart phones or tablets surpass the wildest dreams.

Images as visual aid

Moving pictures help to learn basic math through movements, plus they are far better support for the visualization of math concepts than static ones.

Search on *Giphy* for math pictures. Examples of pictures that be used to learn basic multiplication can be found at http://giphy.com/gifs/cute-kids-family-ATfzy53tX2DFS

The images can be downloaded and embedded, so they can be very useful parts of any home created learning materials.

Teach with Fergy blog is a great source of math gifs: http://www.teachwithfergy.com/21-gifs-that-explain-mathematical-concepts

Mathwarehouse provides a lot math teaching aids including downloadable images: http://www.mathwarehouse.com

GeoGebra for visualization

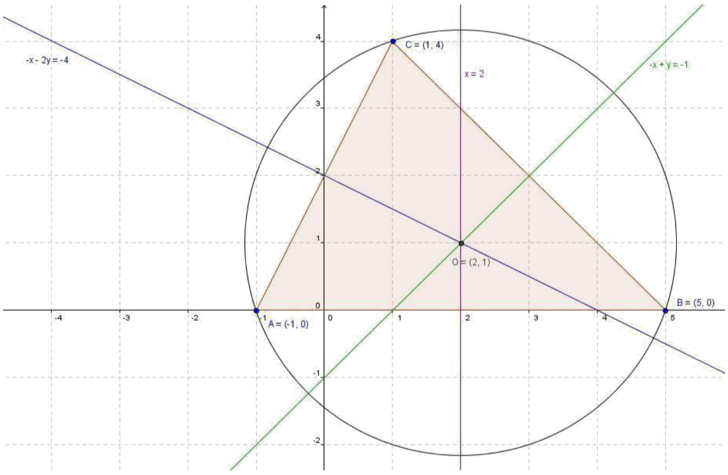
GeoGebra is an interactive application, intended for learning and teaching mathematics and science from primary school to university level. Geometry, algebra, statistics and calculus can be taught through this tool.

GeoGebra is available for Windows, Mac IOS and Linux, and with its tablet apps for Android and iPad. It is also a web application based on HTML5 technology.

Combining real-life math tasks and technology

A math task in the school: A (-1, 0), B (5, 0), C (1, 4) are the coordinates of the vertices of a triangle. Write the equation of the circle that can be wrapped around a triangle.

Converted to a real-life task: Three coordinate points are given A (-1, 0), B (5, 0), C (1, 4). Consider the three points the coordinates of three trees in a garden. The owner of the garden bought a goat. Where should he tie the goat if he wants the goat graze the grass in a circle surrounding the trees?



Here is the figure with the centre of the circle around a triangle. The figure was created using GeoGebra. The program provides an opportunity to make visible these linear equations for the learners. The programme has options to play the steps so that way the process can be visualized.

Good maths requires experience

Having activity and getting experiences are key to building any skills. Math is a tricky subject, because one can easily get an understanding, but it is not enough to be good at math.

There are tremendous math games that help to build math concepts and keep the learner in practice. Gamification is a good way to involve learners in more hard work, and math is a subject that is easy to gamify. Gamification creates a good learning environment by capturing the learner’s attention, giving instant feedback and an optimal challenge.

Some examples:

- *Math Playground* provides a wide offer on systematically arranged math topics: http://www.mathplayground.com
- *Math Is Fun* is a similar website: https://www.mathsisfun.com

Math problem solvers

The math solver programs and applications are useful to check the solution or to find solution to math tasks. There are simply solvers, like http://www.cymath.com.

Again, it is worth to visit the *Mathwarehous*. On the http://www.mathwarehouse.com there is also an ultimate math solver from basic math through trigonometry till calculus.

Wolframalpha is an online platform where with a minimal IT knowledge math questions can be entered, and the program will help to solve. The advantage of this site is that it shows the solving process in different ways and means, plus there are many other options to use: http://www.wolframalpha.com

Photomath reads and solves mathematical problems by using the camera of a mobile device in real-time. It makes math easy and simple by educating users on how to solve math problems: https://photomath.net

Reasoning and calculating

"If you want to err hard, use a machine." (Dr. Eva Gyarmathy, http://opina.indra.es/LP/idEN.do)

Using a calculator or any of the wonderful math solvers, one cannot avoid neither translating the task into everyday language nor the control of good sense and estimation.

While we use the technical tools, we use them to assist on tasks that require a lot effort, and don't require human reasoning.

Letting the work on the technical devices doesn't mean to let the mental control and common sense. A result has to be verified and supervised either gained by a machine or by a human brain.

Reasoning is mental power, and up till now no computer could beat us in it. Machines are better than human beings in tasks that can be solved step by step with logic and analysis without fine tuning and creativity.

Videos

- Examples of the visualization in math: https://www.youtube.com/watch?v=y1XXdCm_GEE-Math_Pics_Szolnoki_Waldorf_Iskola.mp4
- Rhythmic movements help to memorize. Example for the use of moving to support the learning of the multiplication table: https://www.youtube.com/watch?v=fyXmdi-oamy4-Multiplication_Table_WithBalls.mp4
- Introduction to Soroban: https://www.youtube.com/watch?v=0tkOYI-6IRM-Soroban_Level_-_1_2.mp4
- Example of the usage of the Soroban in multiplication: https://www.youtube.com/watch?v=3XafnK2gWoQ-Soroban_Multiplication.mp4
- Example of the usage of the technical solutions in combination: https://www.youtube.com/watch?v=FPdJ8LYqNDo-GeoGebra_in_Combination_with_SMART_Notebook.mp4

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